

MODULE ONE

This module addresses the foundational concepts and skills that support all of the Elementary Algebra academic standards.

SC Academic Elementary Algebra Standards included in this module are:

- EA-2.1 Exemplify elements of the real number system (including integers, rational numbers and irrational numbers).
- EA-2.5 Carry out a procedure using the properties of real numbers (including commutative, associative, and distributive) to simplify expressions.
- EA-2.6 Carry out a procedure to evaluate an expression by substituting a value for the variable.
- EA-2.7 Carry out a procedure (including addition, subtraction, multiplication, and division by a monomial) to simplify polynomial expressions.

Lesson # 1
Topic: Writing and Evaluating Algebra Expressions
Standard (s): EA – 2.6

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills related to this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
 - In 6th grade students apply order of operations to simplify whole-number expressions (6-3.2). Students write variable expressions to represent quantities (6-3.3).
 - In Elementary Algebra, students use substitution to find a numerical value for an expression (evaluate). Student understanding should exceed rote operational proficiency.
 - This essential skill is necessary in all subsequent study of mathematics.
- **Taxonomy**
3.1-C
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge
- **Key Concepts**
Evaluation
Simplification
Substitution

II. Teaching the Lesson

Students have been writing algebraic expressions since 6th grade but may need review their conceptual understanding of variables and uses. They have also applied to order of operations to simplify expression and may need a review of this concept. In this lesson, students become fluent in evaluating algebraic expressions (EA-2.6) which an essential skill which is used to check solutions to linear equations, to determine points on a line and to write linear equations.

- **Essential Learning and Understanding**
It is essential for students to do the following for the attainment of this indicator:
 - Understand how variables are used to represent numerical quantities.
 - Substitute values for one or more variables.

- Evaluate algebraic expressions for specified real numerical values.
- Evaluate algebraic expressions that may involve square roots and/or exponents
- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

 - Evaluate $x + y$ for $x = 0.3$, $y = 2/5$
 - Evaluate x^2 for $x = \sqrt{3}$
 - Evaluate $\frac{xy^2z^3}{y}$ for $x = 5$, $y = 3$, and $z = -2$
- **Non-Essential Learning and Understand**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

 - Substitute and evaluate expressions that are not related to the indicators of Elementary Algebra.
 - Substitute and evaluate expressions involving imaginary numbers.
- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

 - Evaluate $\sin x$ for $x = \pi/2$
 - Evaluate $\log x$ for $x = 100$
- **Misconceptions/Common Errors**
 - Students may not recognize xy , $(x)(y)$, and $x \bullet y$, as x multiplied by y .
 - Students may misapply the Order of Operations.
- **Technology**
 - Students may use technology for complex computation.
 - Students may use the STO on TI graphing calculators to evaluate expressions.

III. Assessing the Lesson

Assessment Guidelines: The objective of this indicator is to carry out a procedure to evaluate an expression by substituting values for variables. Therefore, the primary focus of the assessment should be for students to carry out such procedures.

- **Assessment Item Example:**

- If $a = 3$ and $b = -2$, what is the value of the expression

$$\frac{2a + ab^3}{7(a - 5b)} \quad ?$$

A. $\frac{-210}{31}$

B. $\frac{-210}{91}$

C. $\frac{-18}{31}$

D. $\frac{-18}{91}$

- The area of a circle is given by the formula $A = \pi r^2$. Find A when
when $r = 3$?

A. 3π

B. 6π

C. 12π

D. 36π

- The formula for finding the perimeter of a square is $P = 4s$. Find P
when $s = 5$.

A. 9

B. 4

C. 20

D. 16

IV. Resources

Lesson # 2
Topic: The Real Number Line and Operations with Integers
Standard (s): EA – 2.1

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills related to this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
 - In 6th grade, students applied order of operations (6-3.2). In 8th grade students compare rational and irrational numbers by using the symbols \leq , \geq , $<$, $>$, and $=$ (8-2.4). Also, students apply an algorithm to add, subtract, multiply, and divide integers (8-2.1).
 - In Elementary Algebra students find examples of the following subsets of the real numbers: integers, rational numbers, and irrational numbers.
 - In Intermediate Algebra, students build on their knowledge of the real number system by studying complex numbers. Intermediate Algebra students carry out a procedure to simplify expressions involving powers of i (IA-3.1). In addition, Intermediate Algebra students carry out a procedure to perform operations with complex numbers (including addition, subtraction, multiplication, and division) (IA-3.2).
- **Taxonomy**
2.2-B
Cognitive Process Dimension: Understand
Knowledge Dimension: Conceptual Knowledge
- **Key Concepts**
Real Numbers
Integers
Rational Numbers
Irrational Numbers

II. Teaching the Lesson

In this lesson, students not only focus on the four arithmetic operations but also transfer their understanding of the whole number system to integers, rational and irrational numbers. The real number line is an example tool for demonstrating the relationship among the types of numbers. Students performed operations with integers in 8th grade but may require a short review of this concept.

- **Essential Learning and Understanding**

It is essential for students to do the following for the attainment of this indicator:

- Give examples of integers, rational numbers, and irrational numbers.
- Understand how integers, rational numbers, and irrational numbers are interrelated.
- Classify numbers as integers, rational numbers, or irrational numbers.
- Perform the four arithmetic operations with integers

- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- Identifying 5 is an integer and a rational number.
- Provide an example of a number that is an integer and a rational number, such as -7.
- Identify $0.\overline{66}$ as a rational number.
- Give an example of a rational number such as $-\frac{3}{8}$.
- Identify 0.25 as a rational number.
- Provide an example of an irrational number, such as 0.676676667... (does not repeat in a pattern or terminate) or π .
- Identify $\sqrt{2}$ as an irrational number.
- Explain why $\sqrt{7}$ is irrational. (A formal proof is not required).
- $5 + (-15)$
- $\frac{-20}{-10}$
- $9 \cdot -5$

- **Non-Essential Learning and Understand**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

- Know additional subsets of the real number system (whole numbers and counting numbers).
- Study imaginary numbers.
- Study complex numbers.

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

- Identify 7 as a counting number.
- Know that infinitely many irrational numbers are between 0 and 1 on the number line.

- Give an example of a complex number, such as $5 + 3i$.
- **Misconceptions/Common Errors**
Students may not identify integers as a subset of the rational numbers. Therefore, students may fail to classify integers as rational numbers.
- **Technology**
Students may use technology to identify the existence of a pattern when calculating a quotient.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to exemplify elements of the real number system. Therefore, the primary focus of the assessment should be for students to give examples of integers, rational numbers, and irrational numbers. Because this is conceptual knowledge, assessments should test the student's ability to apply this concept to any integer, rational number, or irrational number, not to be restricted to memorized examples. Students should understand the interrelationships among integers, rational numbers, and irrational numbers.*

In addition to exemplify, students should be able to:

Classify numbers as integers, rational numbers, or irrational numbers.

- **Assessment Item Examples**
 - Which number is not rational?
 - A. 4
 - B. 0.3
 - C. $\sqrt{3}$
 - D. 6.17
 - Which number is rational?
 - A. 2.151551555...
 - B. $7.\overline{07}$
 - C. $\sqrt{2}$
 - D. π
 - Non-repeating, non-terminating decimals are always
 - A. Irrational
 - B. Rational
 - C. Natural
 - D. Integers
 - The number -4 is which of the following?
 - A. An irrational number

- B. A natural number
- C. An integer
- D. A positive number

IV. Resources

Lesson # 3
Topic: Real Number Properties
Standards (s): EA – 2.5

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills related to this standard. It is recommended that students are pre-assessed on this prior knowledge of these properties..

- **Continuum of Knowledge**
 - In 8th grade students use commutative, associative, and distributive properties to examine the equivalence of a variety of algebraic expressions (8-3.3).
 - In Elementary Algebra students carry out a procedure using the properties of real numbers (including commutative, associative, and distributive) to simplify expressions.
 - This essential skill is necessary in all subsequent study of mathematics
- **Taxonomy**
3.1-C
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge
- **Key Concepts**
Properties of real numbers
Commutative Property
Associative Property
Distributive Property
Expressions
Terms

II. Teaching the Lesson

In this lesson, students gain an understanding of commutative, associative and distributive properties by applying them to simplify algebraic expressions. Students also gain a deeper conceptual understanding of the concept of equivalency. Students will continue to use these properties, in the next lesson, to address indicator EA – 2.7 (carry out addition and subtraction to simplify polynomial expressions).

- **Essential Learning and Understanding**
It is essential for students to do the following for the attainment of this indicator:
 - Use one or more of the following properties of the real number system to simplify expressions: Commutative, Associate, and Distributive.
 - Simply algebraic expressions, including numerical expression.
- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- Which expression is equivalent to $5(3x-2y)$?
 - a. $15x-2y$
 - b. $3x-10y$
 - c. $15x-10y$
 - d. $5xy$
 - e. $-30xy$
- Simplify $-5 + (3m - 2)m$
- Simplify $4(2 + x) + (1 - 3x) + 7$
- Simplify $2\pi + 5\pi$
 - a. Solution: $2\pi + 5\pi = (2 + 5)\pi = 7\pi$
- Simplify $\sqrt{2} + 3\sqrt{2} = (1 + 3)\sqrt{2} = 4\sqrt{2}$
 - a. Solution: $\sqrt{2} + 3\sqrt{2} = (1 + 3)\sqrt{2} = 4\sqrt{2}$

- **Non-Essential Learning and Understand**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

- Identify which property is used to justify equivalent expressions.
- Demonstrate knowledge of additional properties of the real number system beyond the Associative, Commutative, and Distributive Property.

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

- $a(x + y) = ax + ay$ is true by what property?
- $(m + n) + p = m + (n + p)$ according to what property?
- $7 = 7$ according to what property?
- Create an example of the Commutative Property of real numbers using symbols or numbers

- **Misconceptions/Common Errors**

Students may not properly apply the distributing property stating that $3(x + 2) = 3x + 2$ instead of the correct answer, which is $3(x + 2) = 3x + 6$.

- **Technology**

Students may use technology to verify equivalent expressions by substituting values and/or using computer algebra systems (CASs).

III. Assessing the Lesson

Assessment Guidelines: The objective of this indicator is to carry out a procedure using the properties of real numbers (including commutative, associative, and distributive) to simplify expressions. Therefore, the primary focus of the assessment should be for students to carry out such procedures.

• **Assessment Item Examples**

- Simplify: $3(2x - 5)$
 - A. $6x - 5$
 - B. $6x - 15$
 - C. $2x - 15$
 - D. $\frac{2}{3}x - \frac{5}{3}$
- Which expression is equivalent to $3(x + 4) + 2(x - 3)$?
 - A. $5x + 1$
 - B. $5x + 9$
 - C. $5x + 6$
 - D. $x + 1$
- Simplify: $4(x - 7) - 2(2 - 3x)$
 - A. $-2x + 24$
 - B. $10x - 32$
 - C. $x - 32$
 - D. $10x - 24$
- Simplify: $5\sqrt{3} + \sqrt{3}$
 - A. $6\sqrt{3}$
 - B. $5\sqrt{3}$
 - C. $4\sqrt{3}$
 - D. $15\sqrt{3}$

IV. Resources

- **Activity:** Have students create models of the properties using objects or pictures instead of numbers. They can cut out pictures from magazines or draw them. For example, $\heartsuit + \diamondsuit = \diamondsuit + \heartsuit$.

Lesson # 4
Topic: Combining like terms
Standards (s): EA – 2.7

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
 - In 8th grade students apply order of operations to simplify whole-number expressions (6-3.2). Apply an algorithm to add, subtract, multiply, and divide integers (8-2.1). Understand equivalent symbolic expressions as distinct symbolic forms that represent the same relationship (6-1.4, 7-1.4, and 8-1.4). Use commutative, associative, and distributive properties to examine the equivalence of a variety of algebraic expressions (8-3.3).
 - In Elementary Algebra students carry out a procedure (including addition and subtraction) to simplify polynomial expressions.
 - In Intermediate Algebra, students build on their knowledge of the real number system by studying complex numbers. Intermediate Algebra students carry out a procedure to perform operations (including multiplication, exponentiation, and division) with polynomial expressions (IA-4.1). Also, students carry out procedures to perform operations on polynomial functions (including $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \cdot g(x)$, and $f(x)/g(x)$) (IA-2.5).
- **Taxonomy**
3.1-C
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge
- **Key Concepts**
Expressions
Terms
Monomial
Polynomial

II. Teaching the Lesson

In this lesson students become fluent in procedures to simplify polynomial expression using addition and subtraction. Students use their knowledge of operations with integers, real number properties and order of operations to simplify expressions.

- **Essential Learning and Understanding**
It is essential for students to do the following for the attainment of this indicator:
 - Real number properties
 - Add and subtract polynomial expressions.

- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- $(x + 5) + (2x + 3)$
- $(2x^3 + 5x^2 - 3) + (-4x^3 + 9x^2 - 1)$
- $(3x^4 - 8x^3 + 2x^2 - 3) + (6x^3 - 2x^2 + 5)$
- $(x + 5) - (2x + 3)$
- $(2x^3 + 5x^2 - 3) - (-4x^3 + 9x^2 - 1)$
- $(3x^4 - 8x^3 + 2x^2 - 3) - (6x^3 - 2x^2 + 5)$

- **Non-Essential Learning and Understanding**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

None Noted

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

None Noted

- **Misconceptions/Common Errors**

When subtracting two polynomials students may forget to distribute the negative thus subtracting only the first term of the subtrahend rather than the entire polynomial provided that the polynomial has more than one term. (minuend – subtrahend = difference)

- **Technology Note**

Students may use computer algebra system technology, which is capable of performing symbolic manipulations, to verify solutions.

III. Assessing the Lesson

Assessment Guidelines The objective of this indicator is to carry out a procedure (including addition and subtraction) to simplify polynomial expressions. Therefore, the primary focus of the assessment should be for students to carry out such procedures.

- **Assessment Item Examples**

- What is $(x^2 + 3x + 4) - (3x^2 + x - 1)$?
 - A. $4x^2 + 4x + 3$
 - B. $-2x^2 + 2x + 5$
 - C. $-2x^2 + 4x + 3$

D. $4x^2 + 2x + 3$

- What is $(x^4 - 3x^2 + 7) + (2x^3 - 1)$?
 - A. $x^4 + 2x^3 - 3x^2 + 6$
 - B. $3x^7 - 3x^2 + 6$
 - C. $x^4 + 2x^3 - 3x^2 + 8$
 - D. $3x^7 - 3x^2 + 8$

IV. Resources

Activity: Students may have difficulty determining which terms are alike. Using different colored markers to identify like terms or writing the terms on different colored squares may help. For example, all the x^2 terms can be on red squares, the constants on blue squares and the x terms on white squares. Students sort them by color and then discuss other common characteristics of other than color such as they all have a square on the x .

Lesson # 5
Topic: Operations with matrices
Standard (s): EA – 2.9

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
 - In 8th grade students organize data in matrices or scatterplots as appropriate (8-6.2).
 - In Elementary Algebra students carry out a procedure to perform operations with matrices (including addition, subtraction, and scalar multiplication).
 - In Geometry, students build apply transformations (including translation and dilation) to figures in the coordinate plane by using matrices (G-6.4).
- **Taxonomy Level**
3.1-C
Cognitive Process Dimension: Apply
Knowledge Dimension: Procedural Knowledge
- **Key Concepts**
Matrix
Row
Column
Dimension (size of a matrix)
Matrix addition
Matrix subtraction
Scalar
Scalar multiplication

II. Teaching the Lesson

In this lesson, students apply their knowledge of operations with integers and matrices to perform operations with matrices. Although students have organized data into matrices in the eighth grade, a review of this concept will be useful in setting the context for operations with matrices.

- **Essential Learning and Understanding**
It is essential for students to do the following for the attainment of this indicator:
 - Add two matrices of size no larger than a 3x3 matrix.
 - Subtract two matrices of size no larger than a 3x3 matrix.
 - Multiply a matrix by a scalar for matrices of size no larger than a 3x3.
- **Examples of Essential Tasks**
These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- Perform the operation or explain why it is not possible.

$$\begin{bmatrix} 7 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 0 \end{bmatrix} - \begin{bmatrix} 3 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 8 & -7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -8 & 7 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -13 \\ 0 & 17 \\ -1 & 20 \end{bmatrix} - \begin{bmatrix} 1 & -4 \\ 6 & -1 \\ 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 0 \\ 4 & 1 & -8 \end{bmatrix} + \begin{bmatrix} 9 & -6 & 1 \\ 7 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 9 & -6 & 1 \\ 7 & 0 & -1 \end{bmatrix} \text{ (Explain why it is not possible to perform the operation.)}$$

$$\begin{bmatrix} 1 & 7 & -6 \\ 9 & 1 & 10 \\ -11 & 12 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 7 & 19 \\ 3 & 0 & 7 \\ -5 & 12 & 1 \end{bmatrix}$$

$$2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$-7 \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$4 \begin{bmatrix} 3 & 11 & -5 \\ -6 & 0 & 10 \\ 8 & 2 & 1 \end{bmatrix}$$

$$-4 \begin{bmatrix} 3 & 11 & -5 \\ -6 & 0 & 10 \\ 8 & 2 & 1 \end{bmatrix}$$

- **Non-Essential Learning and Understanding**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

Perform matrix multiplication.

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

$$\begin{bmatrix} 3 & -1 \\ 8 & 7 \end{bmatrix} \times \begin{bmatrix} -1 & 5 \\ 1 & 3 \end{bmatrix}$$

- **Misconceptions/Common Errors**

Students may forget to distribute the negative to each term in the subtrahend when subtracting matrices. [minuend – subtrahend = difference]

- **Technology Note**

Students may use technology to verify solutions.

III. Assessing the Lesson

Assessment Guidelines: The objective of this indicator is to carry out a procedure to perform operations with matrices (including addition,

subtraction, and scalar multiplication). Therefore, the primary focus of the assessment should be for students to carry out such procedures.

- **Assessment Item Example**

IV. Resources